TechTest2007

Merit Scholarship Examination
in the Sciences and Mathematics
given on 31 March 2007

sponsored by

The Sierra Environmental Studies Foundation

Summary of rules and regulations for the exam:

1. Students will bring their own unmarked blue books to the exam; 3 to 5 blue books should suffice. On the cover of each blue book print your name, today's date '31 March 2007', the name of the exam 'TechTest2007', and the number/totalNumber of the blue book; e.g. the second of three blue books would be labeled 2/3.
2. Students will have four hours to take the test, 9am-1pm. Blue books not handed in by 1pm will not be graded.
3. There will be no passing or exchanging any kind of materials or information between test takers. If there is a problem, contact a proctor.
4. TI and Sanyo calculators may be used and the graphing types, TI 83 through TI-89, are expected.
5. Two, 8.5" X 11" 'cheat sheets' may be used under the following conditions: They are handwritten (both sides if desired) with your name clearly printed on each sheet.
6. No cell phones or communication devices of any kind may be used at any time during the test.
7. Students may leave the test to go to the restroom, one at a time.
8. Juniors will be allowed to come to the test and take it under the rules here stated, but will not be considered for scholarships. The eligibility requirements for Juniors remain the same, 3.0 min gpa, specified majors etc.
9. The determination and announcement of the winners is final. No re-grading or petitioning of any kind is allowed once the winners are determined.
10. All students will do and submit only their own work.

Take your time while answering these questions. You have plenty of time to take a deep breath and relax. Use your bluebooks to calculate and record ALL of your answers. Clearly label each problem and clearly show each step of your calculations. Answer as many questions as you can. Good Luck!

1. Finding a Maximum. For the given function find the maximum value of *y*.

$$y = -2x^2 + 14x$$

2. Balancing a Chemical Equation. Balance the following chemical equation:

$$C_2H_6 + O_2 \rightarrow CO_2 + H_2O$$

3. Work Done By Friction. A friction force that changes as a function of distance is applied to car tires as the car skids. What is the total work done by the friction force between x = 2m and x = 10m?

$$F_f = 1.25x^2 + 100N$$

4. Differential Equation. Solve the differential equation that is used to describe an adiabatic change of state of air for pressure *p*, volume *v* and constant, *c*. Show your answer giving *p* as a function of *v*.

$$p(dv) + cv(dp) = 0$$

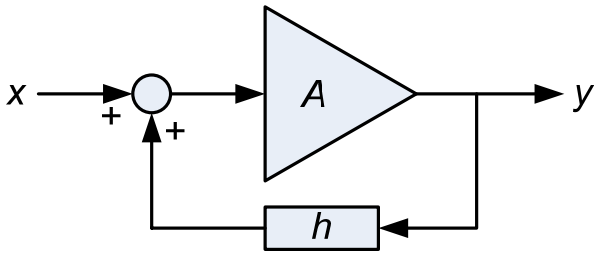
5. Data Interpretation. A substance with an unknown half-life is created in a lab. Use the data to determine:

a. The approximate half-life of the material;

b. An equation that can be used to find the number of grams remaining after a time, *t*.

Time(μs)	Mass(mg)
0	5.00E-02
1	3.13E-03
1.5	7.81E-04
2	1.95E-04
2.5	4.88E-05
3	1.22E-05
3.5	3.05E-06
4	7.63E-07
4.5	1.91E-07
5	4.77E-08

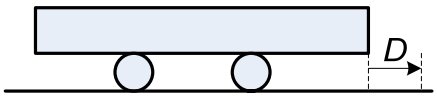
6. Stability and Negative Feedback. Nature and Man use negative feedback to make systems ranging from sun-following flowers to electronic circuits work properly when some of their components (subsystems) degrade. In electronics we can take a portion of the output (y) of an amplifier, reverse its sign, and sum it with the external input (x) to create a more stable amplifier whose performance is less sensitive to the gain changes of the original amplifier. The essence of this system is shown in the diagram below.



Here the 'open-loop' gain of the amplifier is A. The feedback gain is h so that the quantity hy is summed with the x before being input to the amplifier. We see that when $h = 0$ then the open-loop output is just the expected $y = Ax$. We select $0 > h > -1$ to close the loop and create a negative feedback stabilized amplifier that now has a gain A_f with output $y = A_f x$.

a) Derive the formula for A_f as a function of A and h.
b) To obtain a feedback amplifier with $A_f = 50$, specify the nominal value of A given that $h = -0.01$.
c) Derive the (first-order) formula for computing the sensitivity of A_f to changes in A.
d) Using the sensitivity formula, demonstrate the gain in stability of the above feedback amplifier by computing the percent change in A_f when A degrades to 90% of its original value.

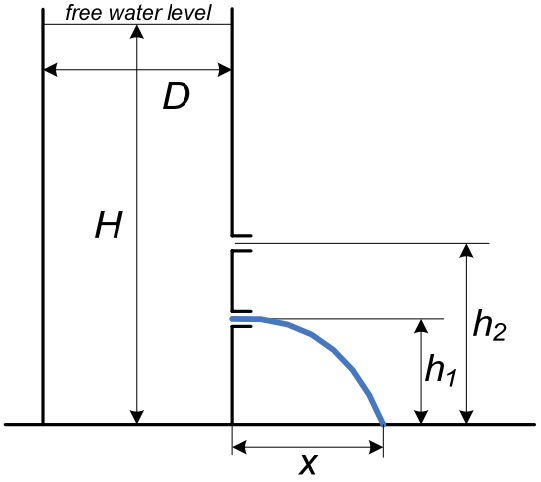
7. Moving A Slab on Rolling Logs. A flat stone slab is moved by placing it on parallel cylindrical logs of identical dimensions as shown in the figure below.



a) Derive the formula for calculating the distance D the slab moves relative to the ground as a function of the number n of revolutions its supporting logs of radius r roll on a flat surface.
b) How many feet forward will the slab move when supported by one foot diameter logs that roll 3.5 revolutions?

8. Minimizing Perimeter to Area Ratio of a Rectangle. Derive the shape, expressed as the ratio l/w, of a rectangle of length l and width w that minimizes P/A, the ratio of its perimeter to area.

9. Gunfighters' Chances for Survival. Three probability impaired gunfighters have decided to settle their differences once and for all by having a three-way duel. Abe and Ben are sure shots who never miss their target, but Clem only manages to hit what he shoots at fifty percent of the time. To give Clem an 'equal chance' they all agree to fire one shot at each other sequentially in a predetermined random order until only one remains standing. Calculate the probability of survival for each of the gunfighters in such a gunfight.



10. Two Holes in a Tank. Is it possible to locate and concurrently open two equal sized holes aligned vertically on the side of a full cylindrical tank filled with water to height *H* (as shown in figure) so that the issuing water streams will strike the ground at the same point? Prove your answer ignoring the effects of air pressure and resistance to motion.